

## Math 155 Worksheet 10

Name:

ID:

1. (Population Growth) A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420. (You can leave your answers in fractional or in log, or in exponential form. There is no need to convert your answers to decimals).

- (a) Find an expression for the number of bacteria after  $t$  hours.
- (b) Find the number of bacteria after 3 hours.
- (c) Find the rate of growth after 3 hours.
- (d) When will the population reach 10,000?

2. (Radioactive decay) The half-life of cesium-137 is 30 years. Suppose we have a 100 mg sample. (You can leave your answers in fractional or in log, or in exponential form. There is no need to convert your answers to decimals).

- (a) Find the mass that remains after  $t$  years.
- (b) How much of the sample remains after 100 years?
- (c) After how long would it take the sample to decay to 20% of its original amount?
- (d) After how long will only 1 mg remain?

**3** Find the exact value of each expression:

(a)  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ .

(b)  $\tan^{-1}(\sqrt{3})$ .

(c)  $\sec(\tan^{-1}(2))$ .

**4.** Suppose that  $F(x) = x \tan^{-1}(f(x))$ . If  $f(3) = 1$  and  $f'(3) = 2$ , find

(a)  $F'(3)$ .

(b) Find an equation of the tangent line to the curve  $y = F(x)$  at the point when  $x = 3$ .

**5.** Find the derivative of these functions.

(a)  $y = e^{\sec^{-1}(x)}$ .

(b)  $y = \sqrt{1-x^2} \arcsin(x)$ .

(c)  $y = \tan^{-1}(x - \sqrt{1+x^2})$ .

**6.** Use logarithmic differentiation to find  $y'$  for  $y = \sqrt{x}e^{x^2}(x^2 + 1)^{10}$ .