MATH 155 – Review Problems for EXAM 2

1. Definitions of Derivatives

(1.1) Use the definition of derivative to compute $f'(a)$, for each of the function $f(x)$ given below.
(a) $f(x) = \sqrt{x+2}$.
(b) $f(x) = \dfrac{x-1}{x-2}$.
(c) $f(x) = \dfrac{2}{x-3}$.
(d) $f(x) = x^2 + 3x + 2$.

(1.2) Each limit below representing a derivative of some function f at some point a. State such an f and a in each case.
(a) $\lim\limits_{h\to 0} \dfrac{\sqrt[4]{16+h}-2}{h}$.
(b) $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\tan(x)-1}{x-\frac{\pi}{4}}$.
(c) $\lim\limits_{x\to 1} \dfrac{x^4+x-2}{x-1}$.

(1.3) Compute the derivative $f'(x)$ by using the definition of derivative.
(a) $f(x) = x^2 - x + 3$,
(b) $f(x) = x + \sqrt{x+2}$.
(c) $f(x) = \dfrac{3}{1+x}$.
(d) $f(x) = \dfrac{3+x}{1-3x}$.

2. Tangent lines, normal lines, velocity and acceleration

(2.1) Find equations of the tangent line and normal line to the curve $y = (1+2x)^2$ at the point $(1, 9)$.

(2.2) Find what values of x does the graph $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent line?

(2.3) Find an equation of the tangent line to the curve $y = x\sqrt{x}$ that is perpendicular to the line $3y = 1 - x$

(2.4) Find an equation of the tangent line to the curve $y = x^2 - 5x + 4$ that is parallel to the line $3x + y = 1$

(2.5) The position function of a moving particle is $s = t^3 - 4.5t^2 - 7t$ (in meters), where the time $t \geq 0$ is in seconds.
(a) When does the particle reach a velocity of 5m/s?

(b) When is the acceleration 0? What is the significance of this value of t?

(2.6) If a stone is thrown vertically upward from the surface of the moon with a velocity of 10 m/s, its height (in meters) after t seconds is $h(t) = 10t - 0.83t^2$.
(a) What is the velocity of the stone after 3 seconds?
(b) What is the velocity of the stone after it has risen 25m?
(c) When does the stone reaches its highest position? What is the velocity of the stone when its position is highest?
(d) When does the stone hit the surface of the moon? What is the velocity of the stone when hits the surface of the moon?

3. Using differentiation rules to compute derivatives

(3.1) Differentiate the functions (that is, find f').
(a) $f(x) = -\pi + \dfrac{x^6}{2} - 3x^4 + x$.
(b) $f(x) = \sin(x) - \pi\cos(x) + \sqrt{30}$.
(c) $f(x) = \dfrac{\sqrt{30}}{x^7}$.
(d) $f(u) = \sqrt{2u} + 3\sqrt{u^7}$.

(3.2) Find f''' for each function f below.
(a) $f(x) = \sin(x) + \cos(x)$.
(b) $f(x) = x^4 + 2x + 1$.
(c) $f(x) = \dfrac{\sqrt{30}}{x}$.
(d) $f(x) = \dfrac{x^4 + x^2 - 1}{\sqrt{x}}$.

(3.3) In each of the problem below, compute the derivative of the given function, and do not simplify your answers.
(a) $f(x) = \sqrt{x}\sin(x)$.
(b) $f(x) = 2\csc(x) + 3\tan(x)$.
(c) $f(t) = t(2\cot(t) - 3\sec(t))$.
(d) $g(x) = \dfrac{4x^2 - 1}{6x^2 - 1}$. (
(e) $F(x) = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$. (
(f) $f(x) = \dfrac{1 - \sin(x)}{1 + \cos(x)}$.
(g) $f(t) = \dfrac{1 - \sec(t)}{\tan t + 1}$.
(h) $g(u) = \dfrac{u^6 - 2u^3 + 3}{u^2}$.

(i) $F(x) = \dfrac{x}{x + \dfrac{5}{x}}$.

(3.4) Find an equation of the tangent line to the curve $y = f(x) = \dfrac{\sqrt{x}}{x+1}$ at the point $(4, 0.4)$.

(3.5) Find an equation of the tangent line to the curve $y = f(x) = (x+1)\cos(x)$ at the point $(0, 4)$.

(3.6) Suppose that $f(3) = 4, g(3) = 2$, $f'(3) = -6$ and $g'(3) = 5$. Find

$$(f+g)'(3), (fg)'(3), \text{ and } \left(\frac{f}{g}\right)'(3).$$

(3.7) How many tangent lines to the curve $y = \dfrac{x}{x+1}$ pass through he point $(1, 2)$? At which points do these tangent lines touch the curve?

(3.8) Find the points on the curve $y = \dfrac{\cos(x)}{2 + \sin(x)}$ at which the tangent is horizontal.

(3.9) In each of the problem below, compute the derivative of the given function, and do not simplify your answers.
(a) $f(x) = (x^2 - 2x + 1)^3$.
(b) $f(t) = \sqrt{1 + \tan(t)}$.
(c) $f(t) = \pi^3 + \cos^3(t)$.
(d) $g(x) = 4\sec(5x) + 5\tan(3x)$.
(e) $F(x) = (1 + 4x)^5(3 + x - x^2)^8$. (
(f) $f(x) = (x^4 - 1)^3(x^3 + 1)^4$.
(g) $f(x) = (x^2 + 1)\sqrt[3]{x^2 + 4}$. (
(h) $g(u) = u^2 \sin(\sqrt{u})$.
(i) $F(x) = \dfrac{x}{\sqrt{4 - 3x}}$.
(j) $f(x) = \left(\dfrac{x^2}{x+1}\right)^5$.
(k) $g(u) = \dfrac{\sin^7(u)}{\cos(u)}$.
(l) $F(x) = x\sin(\dfrac{1}{x})$.
(m) $g(u) = \sin(\sin(\sin(u)))$.
(n) $F(x) = \sqrt{\cos(\sin^2(x))}$
(o) $f(x) = \sin(x)\cos(x)$.
(p) $f(x) = \sin(\cos(x))$.

(3.10) In each of the problem below, find f''.
(a) $f(x) = \sqrt{x^2 + 1}$.
(b) $f(x) = (1 + 2x)^8$.
(c) $f(x) = \tan(3x)$.

(3.11) If $h(x) = \sqrt{4 + 3f(x)}$, where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$.

(3.12) If $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$ and $f'(3) = 6$, find $r'(1)$.

(3.13) If $f(x) = x\cos(x)$, find $f^{(4)}(x)$. From what you have computed to get $f^{(4)}(x)$, can you guess a formula for $f^{(n)}(x)$ for any integer $n \geq 1$?

4. Implicit differentiation

(4.1) In each of the problem below, find dy/dx.
(a) $x^2 - 2xy + y^3 = 1$.
(b) $y^5 + x^2y^3 = 1 + x^4y$.
(c) $1 + x = \sin(xy^2)$.
(d) $y\sin(x^2) = x\sin(y^2)$.
(e) $\sqrt{x+y} = 1 + x^2y^2$.
(f) $\sin(x) + \cos(y) = \sin(x)\cos(y)$.

(4.2) For each problems below, find an equation of the tangent line to the curves at the given point.
(a) $x^2 + 2xy - y^2 + x = 2$ at $(1, 2)$.
(b) $x^2 + xy + y^2 = 3$ at $(1, 1)$.

(4.3) For each problems below, find y''.
(a) $\sqrt{x} + \sqrt{y} = 2$.
(b) $x^4 + xy + y^4 = 3$.

(4.4). Find all the points on $x^2y^2 + xy = 2$ where the slope of tangent line is -1.

(4.5). Find equations of both tangent lines to $x^2 + 4y^2 = 36$ that pass through the point (12,3).