

NAME/ID: \_\_\_\_\_

Section # \_\_\_\_\_

**Important:** You should write your solutions in the space given. If you have to write your solution outside the space provided, you must indicate **clearly** where your solutions are. Failing to do so may result in losing credit points as grader will ignore solutions that are not written in the given space.

**PART 1.** This portion of the test consists of Fill in the Blank questions. Write your answers neatly and legibly in the spaces provided. Some partial credit may be given.

1. (3 points) The value of  $\log_6(27) + \log_6(8)$  is an integer, which is equal to \_\_\_\_\_.
2. (3 points) The derivative of  $f(t) = (\sec^{-1}(x))^2$  is \_\_\_\_\_.
3. (3 points) The value  $\cos^{-1}(\tan(\frac{\pi}{4}))$  equals \_\_\_\_\_.
4. (3 points) The differential of  $f(x) = \tan^{-1}(2x)$  is \_\_\_\_\_.
5. (3 points) The value of  $\lim_{x \rightarrow \infty} \tan^{-1}(x)$  is \_\_\_\_\_.
6. (3 points) A sample of radioactive material decayed to 50% after  $\ln(128)$  years. If  $y(t) = Ae^{-kt}$  represents the mass of the material after  $t$  years, then the value of  $k$  equals \_\_\_\_\_.
7. (6 points) Suppose  $f^{-1}$  is the inverse function of a differentiable function  $f$ . If  $f(3) = 2$  and  $f'(3) = 3$ , then  $f^{-1}(2)$  equals \_\_\_\_\_ and  $(f^{-1})'(2)$  equals \_\_\_\_\_.

**PART 2:** This portion of the test consists of multiple choice problems. No partial credit is given in this section, so work very carefully. *Value: 3 points each*

8. The derivative of  $f(x) = \ln(\cos x)$  is

(A)  $-\frac{1}{\sin x}$  (B)  $-\cos x \ln(\sin x)$  (C)  $\frac{\sin x}{\cos x}$  (D)  $\frac{-\sin x}{\cos x}$

(E)  $\sec x$  (F)  $\frac{\cos x}{\sin x}$  (G)  $\frac{1}{\sin x}$  (H) None of these.

9. Given a circle whose area is increasing at the rate  $4 \text{ cm}^2$  per second, at what rate is the radius increasing when the radius of the circle is  $2 \text{ cm}^2$ ?

(A)  $4 \text{ cm/s}$  (B)  $\frac{2}{\pi} \text{ cm/s}$  (C)  $1 \text{ cm/s}$  (D)  $\frac{1}{\pi} \text{ cm/s}$  (E)  $2\pi \text{ cm/s}$  (F) None of These.

10. The value of  $\lim_{x \rightarrow \infty} \frac{3-4e^x}{2-3e^x}$  is

(A)  $\frac{3}{2}$  (B)  $-\frac{3}{2}$  (C)  $\frac{4}{3}$  (D)  $-\frac{4}{3}$  (E)  $0$  (F)  $-1$  (G)  $1$  (H) None of these.

11. The inverse function of  $f(x) = \frac{3x+2}{4x+1}$  is

(A)  $\frac{3x-2}{4x-1}$  (B)  $\frac{2-x}{4x-3}$  (C)  $\frac{3x-1}{4x-3}$  (D)  $\frac{4x+1}{3x+2}$

(E)  $\frac{2x-3}{x-4}$  (F)  $\frac{x-2}{4x-1}$  (G)  $\frac{4x-1}{3x-2}$  (H) None of these.

12. The derivative of  $f(x) = e^{\cot x}$  is

(A)  $-e^{\csc^2 x}$  (B)  $e^{-\csc^2 x}$  (C)  $-e^{\cot x}$  (D)  $e^{\cot x}$

(E)  $\cot x e^{\cos x}$  (F)  $(-\tan x)e^{\cot x}$  (G)  $(-\csc^2 x)e^{\cot x}$  (H) None of These.

13. The derivative of  $f(x) = e^{\ln x^2}$  is

(A)  $e^{\ln x^2}$  (B)  $2 \ln x$  (C)  $2x$  (D)  $x^2$

(E)  $\frac{\ln x^2}{x}$  (F)  $2x \ln x^2$  (F)  $\frac{1}{x} - e^x \cdot \ln x^2$  (G) None of These.

14. The linear approximation of  $\sqrt[3]{67}$  is equal to

- (A) 4   (B)  $4 - \frac{1}{16}$    (C)  $4 + \frac{3}{16}$    (D)  $4 + \frac{1}{16}$    (E)  $4 + \frac{1}{48}$    (F) None of These.

**PART 3:** This portion of the exam will be graded on a partial credit basis. **Answers without supporting work shown on the paper will receive NO credit.**

15. (5 points each) Use logarithmic differentiation or an alternative method to find the derivative of the functions below.

(a)  $f(x) = \sqrt[4]{\frac{\sin(x)}{e^x + x^3}}$ .

(b)  $f(x) = (\cos(x))^{\ln(x)}$ .

16. (5 points each) Find the derivative of each of the functions, do not simplify your solutions.

(a)  $f(x) = \sqrt{x} \ln(x^2 + 1)$ .

(b)  $f(x) = \frac{e^{\tan(x)}}{\sin(x)}$ .

(c)  $f(x) = e^{\sin(x) + \ln(x^3)}$ .

17. (10 points) Find an equation of the tangent line to the curve  $3y = e^{xy-7x} + 2$  at the point  $(0, 1)$ .

18. (5 points each) Find the derivative of each of the functions, do not simplify your solutions.

(a)  $f(x) = x^2 \sin^{-1}(6x)$ .

(b)  $f(x) = \left( \frac{\tan^{-1}(x)}{x} \right)^2$ .

19. (10 points) A cup has the shape of a cone with height 100 inches and radius 30 inches (at the top). If water is poured into the cup at rate of  $9 \text{ in}^3/\text{s}$ , how fast is the water level rising when the water is 2 inches deep? (Hint: Volume of a cone with base radius  $r$  and height  $h$  is  $V = \frac{\pi}{3}r^2h$ ).

## Math 155 Exam Grade Sheet

Question	Grade	
1 – 7	_____	(out of 24)
8 – 14	_____	(out of 21)
15	_____	(out of 10)
16	_____	(out of 15)
17	_____	(out of 10)
18	_____	(out of 10)
19	_____	(out of 10)
<b>TOTAL</b>	_____	(out of 100)