Two functions $f$ and $g$ are said to be **identically equal** if

$$f(x) = g(x)$$

for every value of $x$ for which both functions are defined. Such an equation is referred to as an **identity**. An equation that is not an identity is called a **conditional equation**.

**Quotient Identities**

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

**Reciprocal Identities**

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

**Pythagorean Identities**

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

**Even-Odd Identities**

$$\sin(-\theta) = -\sin \theta \quad \csc(-\theta) = -\csc \theta$$

$$\cos(-\theta) = \cos \theta \quad \sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta \quad \cot(-\theta) = -\cot \theta$$
Establish the identity:
\[
\sin \theta \csc \theta - \cos^2 \theta = \sin^2 \theta
\]
\[
\sin \theta \csc \theta - \cos^2 \theta = \sin \theta \cdot \frac{1}{\sin \theta} - \cos^2 \theta
\]
\[
= 1 - \cos^2 \theta
\]
\[
= \sin^2 \theta
\]

Establish the identity:
\[
\frac{\cos \theta + 1}{\cos \theta - 1} = \frac{1 + \sec \theta}{1 - \sec \theta}
\]
\[
\frac{1 + \sec \theta}{1 - \sec \theta} = \frac{1 + \frac{1}{\cos \theta}}{1 - \frac{1}{\cos \theta}} = \frac{1 + \frac{1}{\cos \theta}}{\frac{1 - \frac{1}{\cos \theta}}{\cos \theta}}
\]
\[
= \frac{\cos \theta + 1}{\cos \theta - 1}
\]

Establish the identity:
\[
\frac{1 + \sin \theta}{1 - \sin \theta} - \frac{1 - \sin \theta}{1 + \sin \theta} = 4 \tan \theta \sec \theta
\]
\[
\frac{1 + \sin \theta}{1 - \sin \theta} - \frac{1 - \sin \theta}{1 + \sin \theta}
\]
\[
= \frac{(1 + \sin \theta)(1 + \sin \theta)}{1 - \sin^2 \theta} - \frac{(1 - \sin \theta)(1 + \sin \theta)}{1 - \sin^2 \theta}
\]
\[
= 1 + 2 \sin \theta + \sin^2 \theta - 1 - 2 \sin \theta + \sin^2 \theta
\]
\[
= \frac{1 + \sin^2 \theta - 1 - \sin^2 \theta}{1 - \sin^2 \theta}
\]


Guidelines for Establishing Identities

• It is almost always preferable to start with the side containing the more complicated expression.
• Rewrite sums or differences of quotients as a single quotient.
• Sometimes rewriting one side in terms of sines and cosines will help.
• Always keep your goal in mind.
\[ 3 \sin^2 \theta + 4 \cos^2 \theta = 3 + \cos^2 \theta \]

\[ \tan^2 \theta \cos^2 \theta + \cot^2 \theta \sin^2 \theta = 1 \]

\[ 1 - \frac{\sin^2 \theta}{1 + \cos \theta} = \cos \theta \]