To solve an oblique triangle means to find the lengths of its sides and the measurements of its angles.

Theorem  Law of Sines
For a triangle with sides $a,b,c$ and opposite angles $\alpha,\beta,\gamma$, respectively,

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

We use the Law of Sines to solve CASE 1 (SAA or ASA) and CASE 2 (SSA) of an oblique triangle. The Law of Cosines is used to solve CASES 3 and 4.

CASE 3: Two sides and the included angle are known (SAS).
CASE 4: Three sides are known (SSS).
Theorem  Law of Cosines
For a triangle with sides \(a, b, c\) and opposite angles \(\alpha, \beta, \gamma\), respectively.

\[ c^2 = a^2 + b^2 - 2ab \cos \gamma \]
\[ b^2 = a^2 + c^2 - 2ac \cos \beta \]
\[ a^2 = b^2 + c^2 - 2bc \cos \alpha \]

In \(c^2 = a^2 + b^2 - 2ab \cos \gamma\), what value for \(\gamma\) gives the Pythagorean theorem? (1) 0 \(\circ\) (2) 30 \(\circ\) (3) 45 \(\circ\) (4) 60 \(\circ\) (5) 90

Solve the triangle: \(b = 3, c = 4, \alpha = 40^\circ\) (SAS)

\[ a^2 = b^2 + c^2 - 2bc \cos \alpha \]
\[ a^2 = 3^2 + 4^2 - 2(3)(4)\cos 40^\circ \]
\[ a^2 = 6.614933365 \]
\[ a \approx 2.57 \]
\( b^2 = a^2 + c^2 - 2ac \cos \beta \)

\[
\cos \beta = \frac{a^2 + c^2 - b^2}{2ac} = \frac{6.6149 + 4^2 - 3^2}{2(2.57)(4)} \approx 0.6622
\]

\( \beta \approx 48.5^\circ \)

\( \gamma = 180^\circ - 40^\circ - 48.5^\circ \quad \gamma = 91.5^\circ \)

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Solve the triangle: \( a = 3, b = 5, c = 7 \) (SSS)

\[
\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}\]
\[
\cos \alpha = \frac{5^2 + 7^2 - 3^2}{2(5)(7)} = \frac{53}{70} \approx 0.7571
\]

\( \alpha \approx 218^\circ \)

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\[
\cos \beta = \frac{a^2 + c^2 - b^2}{2ac} = \frac{3^2 + 7^2 - 5^2}{2(3)(7)} = \frac{33}{42} = \frac{11}{14}
\]

\( \beta \approx 38.2^\circ \)

\( \gamma = 180^\circ - 21.8^\circ - 38.2^\circ = 120^\circ \)