

MATH 124 COLLEGE ALGEBRA LAB 10
Linearization: Modeling AIDS

OBJECTIVES

- Use Linearization to Determine a Modeling Equation for Deaths Due to AIDS
- Compare and contrast Power Models versus Exponential Models
- Use the Logarithmic Function

MATHEMATICAL CONTEXT (Chapter 4: Sections 4.5)

Determining what function should be selected to model a given set of data can be difficult. The scatter plot of a data set that is curvilinear may appear to be quadratic, cubic, or even exponential in nature. However, if the data set displays a linear trend it is easy to see and model. In this lab we explore using logarithmic functions to convert curvilinear data into linear data, which is easy to model. The process of straightening out curvilinear data is called **linearization**. We will linearize two basic classes of functions:

Power Functions: $f(x) = a \cdot x^n$ which include the basic quadratic function $y = a \cdot x^2$ and the basic cubic function $y = a \cdot x^3$

Exponential Functions: $h(x) = a \cdot e^{kx}$ where e is the natural base – since all exponential functions can be converted to base e we will focus on only this exponential model.

AIDS Epidemic



Many epidemics grow exponentially until they are contained or run through a large proportion of the population. Is the growth of Acquired Immunodeficiency Syndrome (AIDS) modeled by an exponential function or a power function? This is the primary question investigated in this lab. We obtained data on the number of (cumulative) AIDS deaths in the United States by year from the U.S. Center for Disease Control (Table 1).

Table 1: AIDS Data

| | | | | |
|-------------|---------|---------|---------|---------|
| End of Year | 1980 | 1981 | 1982 | 1983 |
| AIDS Deaths | 30 | 160 | 626 | 2,137 |
| End of Year | 1984 | 1985 | 1986 | 1987 |
| AIDS Deaths | 5,663 | 12,659 | 24,842 | 41,330 |
| End of Year | 1988 | 1989 | 1990 | 1991 |
| AIDS Deaths | 62,574 | 90,628 | 122,464 | 159,570 |
| End of Year | 1992 | 1993 | 1994 | 1995 |
| AIDS Deaths | 201,419 | 247,152 | 297,809 | 349,223 |

Exercise 1: Use the *Grapher* to plot the data. Select **Options, Spread Sheet** to open the data

table. Select **Edit, Add Rows** and enter 16 to get 16 rows for the 16 data points. Enter the data letting 1980 be year 1, 1981 be year 2, etc. Plot the data by selecting **Edit, Plot Columns** and selecting x to be x and y to be y.

Use the **Reset Icon**  and the **x-axis Zoom-in Icon**  to view the data. Make a prediction of the model for the data based on the trend of the data. Can you determine from this graph if the model is a power function or an exponential function? Explain your choice.

Exponential Model for AIDS


Since many epidemics grow exponentially, let's test the conjecture that the model for the AIDS epidemic is exponential by linearizing the data. We discussed linearizing data in class by taking the logarithm of the x values and the logarithm of the y-values and plotting $(x, \ln(y))$ to get the semi-log plot or plotting $(\ln(x), \ln(y))$ to get the log-log plot.

Exercise 2: If we guess that the data is exponential in nature, which of the following methods do we use to linearize the data?

1. Examine the log-log plot of the data to see if it is linear.
2. Examine the semi-log plot of the data to see if it is linear.

So let's test to see if the data is modeled by an exponential function by implementing the appropriate method from Exercise 2.

Exercise 3: First we use the *Grapher* to find the natural logarithm of all the x and y values. Return to the data table by selecting **Options, Spread Sheet**. Select **Edit, Add Column, Transformation** and enter $\ln(x)$, then select OK. NOTE that we are using the natural logarithmic function represented in *Grapher* by $\ln(x)$. Repeat this process to find the natural logarithm of all the y-values using $\ln(y)$. Select **Edit, Plot Columns** and choose x to be x, y to be the $\ln(y)$ and select OK to plot the semi-log of the data to determine if it is linear.

Use the **Reset Icon**  to get a better scale for viewing the semi-log data. Does the semi-log plot appear to be linear or curvilinear and what does that tell you about the model being exponential?

Exercise 4: Use the *Grapher* to find a least squares best fit line for the semi-log data by selecting **Options, Least Square Fit**, inputting $y=ax+b$, selecting OK, and entering the default values of a as 1 and b as 1. What is the line of best fit? How good does it fit the semi-log plot? Does it confirm your conclusions from Exercise 3 concerning whether the AIDS data is modeled by an exponential function?

Exercise 5: Use the linear model from Exercise 4 to determine an exponential model for the data. Plot the exponential model with the data. Provide the equation of the model and discuss how it fits the data.

Power Model For AIDS

On the data sheet portion of the lab you will be asked to repeat exercises 2 through 5, but this time to determine a power model for the data.