EXAM III March 31, 2005, NAME:

1. Find the inverse Laplace transform of the following (10 pts) \( \frac{1-2s}{s^2+4s+5} \)

2. Write the complex number \(-1 + \sqrt{3}i\) in the form \(re^{i\theta}\). (7 pts)

3. Find the recurrence relation only by means of a power series about the point \(x_0 = 1\) DO NOT SOLVE. (10 pts)

\[ y'' - xy' - y = 0 \]
4. Find the general solution of the following ODE. (12 pts)

\[ y^{(5)} - 3y^{(4)} + 3y''' - 3y'' + 2y' = 0 \]

5. Solve the given differential equation by means of a power series about the point \( x_0 = 0 \). (12 pts)

\[ y'' + xy' + 2y = 0 \]
6. Find the general solution of the following ODE. (12 pts)

\[ y^{vi} + y = 0 \]

7. Find the general solution of the given ODE using Laplace transforms. (15 pts)

\[ y'' - y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = -1 \]
8. Solve the given system. (12 pts)

\[ x'_1 = 4x_1 - 3x_2 \\
   x_2 = 8x_1 - 6x_2 \]

9. Are the following statements True or False. (12 pts)

i) Given the homogeneous system \( A\vec{x} = \vec{0} \). If the determinant of A is non-zero then there exists infinitely many solutions.

ii) A 5th order linear ODE can be reduced to a system of 5 first order ODE’s.

iii) If \( f(t) \) is continuous on the interval \((0, \infty)\) then the Laplace transform of \( f(t) \) always exists.

iv) A power series solution to a second order ODE can be thought of as an approximation to the actual solution.

v) The eigenvectors found in problem (8) are linearly dependent.